Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 1, 2003*
Revised: *November 20, 2003*
Accepted: *December 15, 2003*

Scalar brane backgrounds in higher order curvature gravity

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Abstract: We investigate maximally symmetric brane world solutions with a scalar field. Five-dimensional bulk gravity is described by a general lagrangian which yields field equations containing no higher than second order derivatives. This includes the Gauss-Bonnet combination for the graviton. Stability and gravitational properties of such solutions are considered, and we particularily emphasise the modifications induced by the higher order terms. In particular it is shown that higher curvature corrections to Einstein theory can give rise to instabilities in brane world solutions. A method for analytically obtaining the general solution for such actions is outlined. Generically, the requirement of a finite volume element together with the absence of a naked singularity in the bulk imposes fine-tuning of the brane tension. A model with a moduli scalar field is analysed in detail and we address questions of instability and non-singular self-tuning solutions. In particular, we discuss a case with a normalisable zero mode but infinite volume element.

Keywords: Extra Large Dimensions, Cosmology of Theories beyond the SM.

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JHEP12(2003)029

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1. Introduction

Developments in string theory suggest that matter and gauge interactions (described by open strings) may be localised on a brane, embedded into a higher dimensional spacetime. Fields represented by closed strings, in particular gravity, propagate in the whole of spacetime. Such an idea provides an interesting framework to address theoretical issues of particle physics and cosmology, such as the hierarchy and cosmological constant problems, or the source of dark energy and dark matter. Several toy models were introduced (see for example [1, 2, 3] for early works) to address some of these problems. Some directly emerged from string theory setups [4] while others were merely inspired by certain aspects of string theory. The possibility of large extra dimensions was considered in ref. [5] for an unwarped spacetime. It was later shown [6] that warping of five-dimensional spacetime could lead to localisation of gravity on the brane even though the extra dimension was of infinite proper length. Gravity in such models seems four-dimensional down to very small distances where higher (discrete or continuum) Kaluza-Klein modes take over. Later on several models were proposed where gravity was actually quasi-localised, i.e. four-dimensional at intermediate scales and modified at very large distances (see for example [7, 8, 9]). Such models yield interesting possibilities for the origin of dark energy [10], 'observed' in cosmological experiments. However such models seem generically to suffer from ghosts [11] or strong coupling problems [12].

Let us step back to the case of localised gravity. As we mentioned above, gravity may be localised in the vicinity of a distributional, positive-tension, flat four-dimensional brane, embedded in a five-dimensional anti-de Sitter bulk, with an infinite extra dimension. The presence of a non trivial background is essential here. More precisely, the Einstein equations in this case admit a solution of the form

$$ds^2 = e^{2A(z)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dz^2\,. \tag{1.1}$$

The tension of the brane needs to be fine-tuned relative to the bulk cosmological constant in order to give four-dimensional Poincaré invariance. This fine-tuning of parameters corresponds to the usual cosmological constant problem in this context. Linear perturbations around this background show the existence of a single massless zero-mode, together with a continuum of massive Kaluza-Klein modes. The zero-mode is normalisable (because the volume element $\int e^{2A}dz$ is finite) and localised on the brane. This is interpreted as the usual four-dimensional graviton. Furthermore, the wave-functions of the massive modes are highly suppressed on the brane, so that their contribution to four-dimensional gravity at low energy may be negligible. This was further confirmed, at the linear level, by a rigorous derivation of Newton's law experienced by observers on the brane [13].

The Randall-Sundrum model [6] and its basic features rely on assumptions, such as the localisation of matter on a distributional source (see however [14]), a constant curvature bulk, and General Relativity to describe gravity in higher than 4 dimensions. One would like to know how generic these assumptions are. Since gravity plays such a key role in brane world models, it is certainly interesting to investigate how the usual features are modified by more general gravitational theories and backgrounds which are still compatible at some limit with Einstein's theory and fundamental principles, and are also in accord with string theory low energy effective actions. This is the direction we take in this paper.

Toward this aim we add higher curvature contributions to the usual Einstein-Hilbert and cosmological constant terms in the bulk action. In four dimensions the Einstein-Hilbert term is the only term (apart from the cosmological constant) which yields at most second order derivatives in the field equations. In five dimensions this is no longer true [15], and the quadratic Gauss-Bonnet term

$$\mathcal{L}_{\rm GB} = R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd} \tag{1.2}$$

also has this property. Any other quadratic term will produce higher order derivatives in the field equations, which will give rise to ghosts in the theory. Thus the Gauss-Bonnet combination is the only other sensible curvature term which can be included in the action for gravity (in five dimensions).

The Gauss-Bonnet term is the Euler characteristic in four dimensions just as the Ricci scalar is the Euler characteristic of two dimensions. The similar properties of the two terms are direct consequences of this fact. In an arbitrary number of dimensions the most general action yielding second order field equations has been given by Lovelock [16] (see also [17] for an elegant derivation using differential forms). Brane gravity with the Gauss-Bonnet term in a constant curvature bulk spacetime was first studied in [18].[1] A careful treatment of the

[1]See however [19, 20] for the junction conditions.

graviton fluctuations was given in ref. [19] (see also [20] for the full use of Lovelock theory in arbitrary dimensions). Cosmological evolution and consequences of such a setup have also been studied (see for example [21]–[26]). For some early applications to cosmology see [27].

Another generalisation is to add a scalar field, with a Liouville type potential in the bulk. This may represent the dilaton and/or moduli fields of compactified string theory. In this context, gravitational fluctuations around scalar brane backgrounds have been studied (see e.g. [28], and also [29] for a smooth brane realisation). One of the phenomenological benefits of the inclusion of such a scalar field is an interesting reformulation of the cosmological constant problem in brane world models [30]. One is then able to use the additional degrees of freedom to ensure the existence of a four-dimensional Poincaré invariant solution whatever the values of the brane tension and the bulk cosmological constant. Thus there is no fine tuning of the fundamental parameters of the theory. This is the so-called 'self-tuning' mechanism. Unfortunately scalar field brane worlds with Liouville potentials generically suffer from naked singularities at finite proper distance from the brane, at least when sensible four-dimensional gravity is required [31]. This is true in particular for the self-tuning solutions [32]. The presence of the singularities re-introduces fine-tuning into the problem. This therefore represents a rephrasing, and not a solution, of the cosmological constant problem [33].

Close to these naked singularities, the original effective gravitational theory breaks down, because higher order terms in the lagrangian cannot be neglected anymore. We are thus naturally led to consider higher order curvature terms and, for the same reasons, higher order scalar kinetic terms as well. In this paper we will consider terms which are up to quartic order in derivatives in the action. Such actions arise naturally in the context of low energy effective string theory [34], and are known to lead to interesting properties in four-dimensional cosmology [35] as well as in black-hole physics [36]. Brane worlds with actions of this kind have been previously studied for a metrics of the form (1.1) with $A \propto z$. The basic setup and particular solutions were analysed in detail in ref. [37], and asymptotic behaviour of solutions and self-tuning was discussed in ref. [38] (see also [39]). In ref. [40], an exact solution was found with non trivial scalar field profile, no naked singularity and finite four-dimensional Planck mass. However, the 'self-tuning' mechanism was shown to fail since the solution was fine-tuned much in the same way as in the Randall-Sundrum model. In fact, as explained in section 3, in order to address this latter issue, *general* solutions have to be known analytically. The construction, analysis and stability study of such solutions is one of the aims of this work.

The organisation of the paper is as follows: in the next section, we discuss the general action that we will consider. We demonstrate in particular that the origin of the scalar field (e.g. dilaton or moduli) fixes the coefficients of the higher order terms. We give the coefficients for toroidal Kaluza-Klein compactification. Field equations and boundary conditions for a conformally flat brane background (1.1) are discussed in section 3. Next, in section 4, we study the spin 2 linear gravitational perturbations around an arbitrary conformally flat background. We point out in all generality the essential differences of spin 2 perturbations of Einstein-Gauss-Bonnet with respect to ordinary Einstein gravity. We address in particular the question of gravity localisation on the brane. It is also shown generically that stability requirements are far more stringent in higher curvature brane world setups. This essentially means that Einstein brane worlds can develop order α' instabilities in an effective action approach. Having discussed in a general context background and perturbations, we move on to demonstrate how one finds solutions and their gravitational properties (singularities, localisation of gravity, stability etc). In section 5 we show how to obtain the general conformally flat solution to the field equations and their basic properties. In section 6 we apply our general analysis to the algebraically simplest case where the scalar field comes from the compactification of a flat sixth dimension. It is shown that the general maximally symmetric brane world solution is obtained from the corresponding six-dimensional black-hole solution. We concentrate on the cases without naked singularities in the bulk and study their stability and self-tuning requirements. It is intriguing that in the sole case where 'self-tuning' is actually possible (normalisable graviton zero-mode without naked singularity in the bulk, nor brane tension tuning), a ghost appears in the bulk. The last section contains a summary of our main results and our conclusions.

2. Action

Consider the action describing a scalar field which is conformally coupled to gravity, written in the Jordan (or string) frame, with terms which are up to quartic order in the field derivatives

$$S = \frac{M^3}{2}\int d^5x\sqrt{-g}\,e^{-2\phi}\Big\{R - 4\omega\partial_a\phi\partial^a\phi - 2\Lambda + \\ + \alpha\Big[\mathcal{L}_{\rm GB} + 16a\partial_a\phi\partial^a\phi\Box\phi - 16bG^{ab}\partial_a\phi\partial_b\phi - 16c(\partial_a\phi\partial^a\phi)^2\Big]\Big\}. \tag{2.1}$$

This is the most general action of this order which obeys the rules of Lovelock gravity. This essentially means that the field equations obtained by variation of the scalar field ϕ and the metric tensor $g_{\mu\nu}$ are linear in second order derivatives, and do not feature higher order derivatives. This is an essential and natural hypothesis when studying backgrounds with boundaries and linear perturbations of them. In a string theory context the above frame is referred to as the string frame if ϕ is the dilaton. Otherwise (for example if ϕ is a moduli field) it should be referred to as a Jordan frame.

The constants a, b and c express the higher order terms for the scalar field and its interaction with the Einstein tensor. They will be given specific values later on, depending on the physical nature of the scalar field. ω is the Brans-Dicke parameter. The higher order terms couple via the dimensionfull constant α which is of dimension (length)^2. For string theory it is proportional to the string tension α'. Any terms other than those appearing in the above action (2.1) yield higher than second order derivatives, which always result in the existence of ghosts [41], even around a flat background.

Scalar-tensor theories of gravity are also studied in the Einstein frame, which is related to the Jordan frame by the conformal transformation $g_{ab} \to e^{-4\phi/3}g_{ab}$. The Einstein frame action is

$$S_E = \frac{M^3}{2}\int d^5x\sqrt{-g}\Big\{R - \frac{4}{3}(3\omega+4)\partial_a\phi\partial^a\phi - 2\Lambda\, e^{4\phi/3} + \\ + \alpha e^{-4\phi/3}\Big[\mathcal{L}_{\rm GB} + 16\tilde{a}\partial_a\phi\partial^a\phi\Box\phi - 16\tilde{b}G^{ab}\partial_a\phi\partial_b\phi - 16\tilde{c}(\partial_a\phi\partial^a\phi)^2\Big]\Big\} \tag{2.2}$$

where

$$\tilde{a} = a - 3b + \frac{16}{9}, \qquad \tilde{b} = b - \frac{16}{9}, \qquad \tilde{c} = -2a + \frac{4}{3}b + c - \frac{8}{27}. \tag{2.3}$$

The identifications between the coefficients are easily obtained by comparing the field equations in the two frames (see appendix). Written in this form the action more closely resembles the usual Einstein-Hilbert action.

If the scalar field ϕ plays the role of the dilaton, the parameters a, b and c in eq. (2.1) are constrained in order to reproduce the scattering string amplitudes on shell [34]. For $\omega = -1$ this constraint reads,

$$2a - 2b - c + 1 = 0. \tag{2.4}$$

The case $a = b = c = 1$ exhibits additional symmetry (higher-order extension of T-duality [42], as well as σ-model conformal invariance [43]). In the context of brane world models, it has been considered in ref. [38], while the case $\tilde{a} = \tilde{b} = 0$, $\tilde{c} = 1/27$ has been discussed in detail in refs. [37, 40].

Another interesting case arises from toroidal Kaluza-Klein compactification. Start by considering the $5+N$-dimensional action

$$S^{(5+N)} = \frac{M^3}{2L^N}\int d^5x dX^N \sqrt{-g_{(5+N)}}\Big\{R^{(5+N)} - 2\Lambda + \alpha\mathcal{L}_{\rm GB}^{(5+N)}\Big\} \tag{2.5}$$

and take the N extra dimensions to be flat and compact, with $0 \le X_A < L$ and $A, B = 1, \dots, N$. Then for the metric ansatz

$$ds^2_{5+N} = g_{ab}(x)dx^a dx^b + e^{-4\phi(x)/N}\eta_{AB}dX^A dX^B \tag{2.6}$$

we obtain the same field equations as eq. (2.1) with

$$\omega = -\frac{N-1}{N}, \qquad a = (2\omega+1)\omega, \qquad b = -\omega, \qquad c = -(2\omega+1)\omega^2. \tag{2.7}$$

This is the extension to higher curvature gravity theories of the usual Kaluza-Klein toroidal compactification. The scalar field in this case arises simply as the unique size or moduli of the N compact extra dimensions. For instance, in the string frame, $N = 1$ is simply $\omega = a = b = c = 0$, and $N = 2$ is $\omega = -1/2$, $a = c = 0$, $b = 1/2$. As N increases the extra terms in the action (2.1) are switched on, adding successive layers of complication. Interestingly the special case $a = b = c = 1$ of eq. (2.4) corresponds to $N \to \infty$ and $\omega = -1$.

3. Field equations

Consider the conformally flat metric,

$$ds^2 = e^{2A(z)}dx^\mu dx^\nu\eta_{\mu\nu} + e^{2B(z)}dz^2. \tag{3.1}$$

Four dimensional spacetime sections are Poincaré invariant, and the single extra dimension (with coordinate label z) is warped. Note that we still have a gauge freedom relating the metric components A, B and ϕ; for example $B = 0$ corresponds to a gaussian normal system. We shall use this gauge freedom to solve the field equations in convenient coordinate systems in sections 5 and 6. The full field equations are listed in the appendix. The (5,5) component of the generalised Einstein equation gives

$$\begin{aligned} & e^{2B}\Lambda - 2\omega\phi'^2 + 6A'^2 - 8\phi' A' + \\ & +4\alpha e^{-2B}\left(-3A'^4 + 24A'^3\phi' - 36bA'^2\phi'^2 + 4a\left(4A'\phi'^3 + \phi'^4\right) - 6c\phi'^4]\right) = 0 \end{aligned} \tag{3.2}$$

(note the absence of second derivatives). Furthermore a linear combination of the remaining two differential equations can be directly integrated to give

$$\begin{aligned} & e^{4A-2\phi-3B}\Big[e^{2B}(2[\omega+1]\phi' + A') - \\ & -4\alpha\left(3A'^3 + 6A'^2\phi' - 6b[2A'^2\phi' + A'\phi'^2] + 4a[2A'\phi'^2 + \phi'^3] - 4c\phi'^3\right)\Big] = -\mathcal{C}\,, \end{aligned} \tag{3.3}$$

where $\mathcal{C}$ is a constant of integration. It is interesting to note that for the Kaluza-Klein like choice of parameters (2.7), this equation factorises,

$$e^{4A-2\phi-B}(2[\omega+1]\phi' + A')\left(1 - 4\alpha e^{-2B}[3A'^2 + 6\omega A'\phi' + 2\omega(2\omega+1)\phi'^2]\right) = -\mathcal{C}\,. \tag{3.4}$$

The integration constant $\mathcal{C}$ will acquire a physical meaning in the case of 1 dimensional toroidal compactification. The above system of equations (3.2)–(3.3) are independent and yield the full set of solutions of eq. (2.1) for a conformally flat spacetime.

The brane contribution to the action in the Jordan/string frame is

$$S_b = -\int d^4x\sqrt{-h}\,T(\phi) \tag{3.5}$$

where T is the brane tension. We can construct a brane world solution by joining together two spacetimes $\mathcal{M}_R$ and $\mathcal{M}_L$. If we choose the coordinates so that $z < 0$ in $\mathcal{M}_L$, $z > 0$ in $\mathcal{M}_R$ and $z = 0$ on the brane, then the brane junction conditions are

$$\left[e^{-B}(-3A' + 2\phi') + 4\alpha e^{-3B}\left(A'^3 - 6A'^2\phi' + 6bA'\phi'^2 - \frac{4a}{3}\phi'^3\right)\right]_L^R = M^{-3}e^{2\phi}T(\phi) \tag{3.6}$$

$$\begin{aligned} & \Bigg[e^{-B}(-4\omega\phi' - 8A') + 4\alpha e^{-3B}\Bigg(8A'^3 - 24bA'^2\phi' + \\ & +16a\left(A'\phi'^2 + \frac{\phi'^3}{3}\right) - 8c\phi'^3\Bigg)\Bigg]_L^R = -\,M^{-3}e^{2\phi}\frac{dT(\phi)}{d\phi} \end{aligned} \tag{3.7}$$

where $[X]_L^R = X_R - X_L$ denotes the jump of X across the brane. It is interesting to note [38] that a linear combination of the above gives a similar equation to (3.3),

$$\begin{aligned} & \Big[e^{-B}(2(\omega+1)\phi' + A') - 4\alpha e^{-2B}\Big(3(A'^3 + 2A'^2\phi') - \\ & -6b(2A'^2\phi' + A'\phi'^2) + 4a(2A'\phi'^2 + \phi'^3) - 4c\phi'^3\Big)\Big]_L^R = \frac{M^{-3}}{2}\frac{d}{d\phi}\left(e^{2\phi}T(\phi)\right). \end{aligned} \tag{3.8}$$

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Thus if the integration constants in (3.3) are such that $[\mathcal{C}]_L^R = 0$ (which corresponds to $\mathcal{C} = 0$ in the Z_2-symmetric case), the r.h.s. of (3.8) must vanish. This constrains the brane tension to be $T(\phi) \propto e^{-2\phi}$, which is the natural choice for the dilaton coupling to NS-branes in heterotic string theory. Such a case was studied in refs. [37, 40]. In ref. [40] in particular all requirements for 'self-tuning' (absence of naked singularities, finite volume element) were verified apart from 'self-tuning' itself. This was precisely due to the fact that the integration constant was set to be zero. Generically the $[\mathcal{C}]_L^R = 0$ solutions always require fine-tuning of the fundamental parameters. We will investigate in detail a case where $\mathcal{C} \neq 0$ in section 6.

4. Brane gravity and stability

In order to investigate gravity as perceived by a four-dimensional observer and stability issues concerning the background solutions we will in this section consider perturbations around the general background (3.1). This will permit us to understand some important characteristics proper to the Einstein-Gauss-Bonnet theory that we are investigating. In this paper we will concentrate on the spin 2 perturbations which correspond to tensorial gravity on the four-dimensional brane world. We will see that the wave equation operator for the graviton modes in Lovelock gravity is closely analogous to that of Einstein gravity (see [51] for a recent discussion and references within). This follows from the fact that the two theories were constructed using similar requirements for their gravitational properties. The junction conditions will be however crucially different.

Start by choosing an axial gauge for the perturbed metric ($\gamma_{a5} = 0$):

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu} + \gamma_{\mu\nu}(x,z))dx^\mu dx^\nu + e^{2B(z)}dz^2 \tag{4.1}$$

and consider a perturbed scalar field $\phi(z) + \varphi(x,z)$, with $\gamma_{\mu\nu}$ and φ small. The first order perturbation to the $(\mu\nu)$-component of the gravitational field equation yields a rather complicated expression,

$$\begin{aligned}\partial_z(p(z)\partial_z[\gamma_{\mu\nu}-\eta_{\mu\nu}\gamma]) - w(z)\Big(2\partial^\sigma\partial_{(\mu}\gamma_{\nu)\sigma} - \Box_4\gamma_{\mu\nu} - \\ -\partial_\mu\partial_\nu\gamma - \eta_{\mu\nu}[\partial^\sigma\partial^\kappa\gamma_{\sigma\kappa} - \Box_4\gamma]\Big) + \\ +\eta_{\mu\nu}(f_1(z)\partial_z^2\varphi + f_2(z)\partial_z\varphi) + f_3(z)[\partial_\mu\partial_\nu\varphi - \eta_{\mu\nu}\Box_4\varphi] = \delta(z)e^{4A}\left(\frac{dT(\phi)}{d\phi} + 2T(\phi)\right)\eta_{\mu\nu}\varphi\end{aligned} \tag{4.2}$$

where indices are raised and lowered with $\eta_{\mu\nu}$, $\Box_4 = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the flat four-dimensional laplacian and f_1, f_2 and f_3 are some given background functions of z that we will not need for what follows. What will be important however, are the functions w and p which are given by,

$$w(z) = e^{2A-2\phi+B}\,[1 + 4\alpha q(z)] + 4\alpha v'(z)\,, \tag{4.3}$$

$$p(z) = e^{4A-2\phi-B}\,(1 - 4\alpha q(z))\,, \tag{4.4}$$

where in turn

$$q(z) = e^{-2B}(A'^2 - 4A'\phi' + 2b\phi'^2)\,, \tag{4.5}$$
$$v(z) = -e^{2A-2\phi-B}(A' - 2\phi')\,. \tag{4.6}$$

We now project out the transverse traceless components of $\gamma_{\mu\nu}$ [44] (see also [28]). Define symbolically the operator $\pi_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu\partial_\nu/\Box_4$, and consider

$$\bar{\gamma}_{\mu\nu} = \left(\pi_{\mu(\sigma}\pi_{\kappa)\nu} - \frac{1}{3}\pi_{\mu\nu}\pi_{\sigma\kappa}\right)\gamma^{\sigma\kappa}\,. \tag{4.7}$$

This satisfies $\partial^\mu\bar{\gamma}_{\mu\nu} = 0$ and $\eta^{\mu\nu}\bar{\gamma}_{\mu\nu} = 0$. Then, applying the projection (4.7) to the perturbation equation (4.2), we obtain the bulk perturbation equation for the tensor modes

$$\partial_z(p(z)\partial_z\bar{\gamma}_{\mu\nu}) + w(z)\Box_4\bar{\gamma}_{\mu\nu} = 0\,. \tag{4.8}$$

Note that the form of the perturbation operator for $\bar{\gamma}_{\mu\nu}$ remains unchanged for $\alpha \neq 0$. In particular it is free from derivatives of higher than second order. What changes however is the interaction with the background i.e. the background dependent functions $w(z)$ and $p(z)$ which are now of higher order in derivatives than when $\alpha = 0$. This fact is spelt out by the definitions of q and v which are of first order in derivatives. Note in particular the presence of second order derivatives, v', in the definition of w. This will yield important differences to the Einstein case for the perturbed junction conditions on the brane.

Given the spacetime symmetries consider now a plane wave separation of variables:

$$\bar{\gamma}_{\mu\nu}(x,z) = \bar{c}^m_{\mu\nu}(x)\,\psi_m(z) \qquad \text{with } \Box_4\bar{c}^m_{\mu\nu}(x) = m^2\,\bar{c}^m_{\mu\nu}(x) \tag{4.9}$$

where $\bar{c}^m_{\mu\nu}(x)$ is transverse-traceless and m^2 is the mass squared of the graviton mode as perceived by a four-dimensional observer. The scalar interaction wave equation (4.8) now reduces to

$$\left(p(z)\psi'_m\right)' + m^2\,w(z)\,\psi_m = 0 \tag{4.10}$$

valid everywhere in the bulk. From the explicit form of w and p in eqs. (4.3) and (4.4), the associated junction conditions at the brane are given by

$$[e^{-B}\psi'_m]^R_L = 4\alpha[qe^{-B}\psi'_m + e^{-2A-B}m^2\psi_m(A' - 2\phi')]^R_L\,. \tag{4.11}$$

Furthermore, in order for the perturbed bulk to induce a well-defined metric on the brane, the modes ψ_m have to be continuous across the brane.

In order to study the propagation of gravitons (existence of zero mode, stability, and orthogonality of eigenmodes) modulo the initial conditions supplied by the brane we now need to examine the boundary value problem as defined by (4.10) and (4.11). Such problems are generically treated in two ways; either as a Sturm-Liouville boundary problem, or by coordinate transforming to a Schrödinger type equation. Let us follow these methods in turn pointing out the essential differences appearing here due to the higher curvature term corrections.

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If p and w are positive, the l.h.s. of (4.10) is the usual 'self-adjoint' type differential operator with weight function w just like in standard (singular) Sturm-Liouville theory. The boundary conditions for (4.10) are however unusual and call for particular attention. For massless modes $m^2 = 0$, and Z_2 symmetry, (4.11) tells us that $\psi_m' = 0$ for $z = 0$ just like for the $\alpha = 0$ case — a standard Neumann boundary condition on the brane. For the massive modes however this is not the case, we have rather a mixed boundary condition and furthermore the mass of the mode in question appears in the boundary condition itself (4.11). This is precisely due to the presence of higher derivatives in w and is thus a feature of Gauss-Bonnet gravity.[2] We can however treat this in an analogous way by adding a suitable boundary term (total derivative) to the usual scalar product of the eigenvector functional space. Standard Sturm-Liouville considerations will then follow through. Indeed define

$$(\phi, \psi) = \int_{\text{Bulk}} \phi\psi w \, dz + 4\alpha[\psi\phi v]_L^R \tag{4.12}$$

where not only w but also $[v]$ has to satisfy positivity conditions. We have included a non-trivial boundary term which arises from the second order derivative appearing in w (4.3). Suitable Sturm-Liouville boundary conditions are imposed at asymptotic infinity. The problem (4.10)–(4.11) is now self-adjoint. The modes are then orthogonal, form a complete basis and the problem is free of tachyon modes. The particular boundary conditions imposed here (4.11) lead us to demand that $[v]$ is also positive along with w and p.

If on the other hand p is negative, the sign of the kinetic term in an effective action for ψ_m will also be negative. The solution will then have a graviton ghost in the bulk. If w and p have opposite signs the corresponding effective mass term will be negative. In this case the system may have tachyon modes and thus be classically unstable. Note that a negative value of $[v]$ can also permit tachyon modes to exist, even if $w > 0$ (see section 6 for an example). This is due to the mixed boundary conditions (4.11) and essentially means that brane world backgrounds which are stable in Einstein gravity can develop an instability once we allow for α corrections. This amounts to a higher order curvature correction instability. This is the most important consequence of the mixed boundary conditions (4.11) and is independent of the presence of the scalar field in the background. If p and w (and $[v]$) are negative the system will be classically stable although a graviton ghost is problematic at the quantum level. Note that the possibility of negative p or w (or $[v]$) only occurs when the α dependent terms are included in the theory. It should also be noted that even if there are no graviton ghosts or tachyons, there will be analogous issues with the scalar field perturbations (tachyons, ghosts and also radiative corrections), although we will not consider them in this paper.

A necessary condition for gravity to be localised on the brane is that the zero mode solution of (4.10)–(4.11) be normalisable. This is equivalent to asking that

$$\int_{\text{Bulk}} w(z) \, |\psi_0|^2 \, dz < \infty \, . \tag{4.13}$$

[2]Not surprisingly similar boundary conditions also appear in the DGP model [7] with the inclusion of the induced gravity term on the brane.

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The zero mode solution of the bulk equation (4.10) is

$$\psi_0 = c_0 + c_1 \int_{\text{Bulk}} \frac{dz}{p}\,, \tag{4.14}$$

where c_0 and c_1 are constants fixed by the boundary conditions. For a Z_2 symmetric brane world, $c_1 = 0$. If the zero mode ψ_0 is constant the normalisation condition (4.13) amounts to having a finite four-dimensional volume element:

$$\int_{\text{Bulk}} w(z)\, dz\,, \tag{4.15}$$

which includes the α contributions. If the volume element (4.15) is not finite and $p > 0$ there will be no normalisable zero mode. However if we consider spacetimes where p has a different sign on each side of the brane, this argument may be bypassed. If w also changes sign the system may be classically stable, although it will have a ghost on one side of the brane. We describe an explicit example of this in section 6.

It is also useful to recast the wave equation (4.10) in a Schrödinger form in the bulk ($z \neq 0$):

$$\left[-\frac{d^2}{dx^2} + V(x)\right]\Phi_m = m^2\,\Phi_m \tag{4.16}$$

with $dx/dz = \sqrt{w/p}$ and $\Phi_m = (wp)^{1/4}\,\psi_m$. The effective potential is then given by,

$$V(x) = \frac{\frac{d^2}{dx^2}[(wp)^{1/4}]}{(wp)^{1/4}}\,.$$

Note that when $\alpha \neq 0$ this equation is not valid on the brane since the coordinate x is ill defined there. The Schrödinger picture is however interesting when studying the large z behaviour.

5. Conformally flat solutions

We will now give a method for obtaining analytically the general solution of the bulk field equations (3.2)–(3.3). If $\mathcal{C} = 0$ then $e^{-B}A'$ and $e^{-B}\phi'$ are constants and the differential equations reduce to algebraic equations, which are easily analysed. Such solutions have been investigated in [37, 38, 40]. To deal with the more complicated case of $\mathcal{C} \neq 0$, we begin by choosing our variable to be $z = d\phi/dA$ (so in these coordinates, the brane is not necessarily at $z = 0$).

The field equations now take the form,

$$\Lambda + A'^2 e^{-2B}\left[P_0(z) - 4\alpha e^{-2B}A'^2 P_1(z)\right] = 0 \tag{5.1}$$

$$\mathcal{C}e^{2\phi-4A} = -e^{-B}A'\left[Q_0(z) - 4\alpha e^{-2B}A'^2 Q_1(z)\right] \tag{5.2}$$

where

$$P_0 = 6 - 8z - 2\omega z^2\,, \qquad P_1 = 3 - 24z + 36bz^2 - 16az^3 - 2z^4(2a - 3c)\,,$$
$$Q_0 = 1 + 2(\omega + 1)z\,, \qquad Q_1 = 3 + 6(1 - 2b)z + 2z^2(4a - 3b) + 4z^3(a - c)\,.$$

For $\Lambda = 0$, (5.1) is easily solved to give an expression for B

$$B = \ln A' + \frac{1}{2}\ln\frac{4\alpha P_1}{P_0}\,. \tag{5.3}$$

Substituting this into (5.2) provides an expression for $\phi - 2A$ in terms of z

$$\phi - 2A = \frac{1}{2}\ln(P_0Q_1 - Q_0P_1) + \frac{1}{4}\ln\frac{P_0}{P_1^3} - \frac{1}{2}\ln(2\mathcal{C}\sqrt{\alpha})\,. \tag{5.4}$$

Differentiating with respect to z and rearranging this expression gives

$$A' = \frac{1}{4(z-2)}\frac{d}{dz}\left\{2\ln(P_0Q_1 - Q_0P_1) + \ln P_0 - 3\ln P_1\right\}. \tag{5.5}$$

Integrating this gives $A(z)$, which can then be substituted into eqs. (5.3) and (5.4) to find $B(z)$ and $\phi(z)$. It is obvious that the choice of convenient a, b and c can simplify the relevant algebra. A similar approach can be used when $\Lambda \neq 0$, although in this case the expression for B (5.3) will be a solution to the quadratic (5.1), which complicates all the subsequent steps.

The properties of these solutions can be analysed by expressing the above polynomials in terms of their roots. Write $P_0 = 6(1-u_1z)(1-u_2z)$, $P_1 = 3\prod_{i=1}^{4}(1-v_iz)$, $P_0Q_1 - Q_0P_1 = 15\prod_{i=1}^{5}(1-w_iz)$. Some of the u_i, v_i, w_i may be zero, or complex. For example, if $a = b = c = \omega = 0$ all these parameters are zero except $u_1 = 4/3$, $v_1 = 8$, $w_1 = -2$. This case will be discussed in detail in section 6.

Assuming that $z = 2$ is not a root of any of the three polynomials, the expressions (5.3)–(5.4) imply

$$A = \frac{1}{4}\sum_i \frac{u_i}{1-2u_i}\ln|1-u_iz| - \frac{3}{4}\sum_i \frac{v_i}{1-2v_i}\ln|1-v_iz| + \frac{1}{2}\sum_i \frac{w_i}{1-2w_i}\ln|1-w_iz| + \text{cst.} \tag{5.6}$$

$$\phi = \frac{1}{4}\sum_i \frac{1}{1-2u_i}\ln|1-u_iz| - \frac{3}{4}\sum_i \frac{1}{1-2v_i}\ln|1-v_iz| + \frac{1}{2}\sum_i \frac{1}{1-2w_i}\ln|1-w_iz| + \text{cst.} \tag{5.7}$$

and

$$B = \ln A' - \frac{1}{2}\sum_i \ln(1-u_iz) + \frac{1}{2}\sum_i \ln(1-v_iz) + \ln\sqrt{2\alpha}\,. \tag{5.8}$$

It is clear from the above expressions that the points $z = 1/u_i, 1/v_i, 1/w_i$ (real values of z only) are singularities of some kind. We will refer to these as critical points. Note that the metric (and ϕ) are only singular at the point $z = \pm\infty$ if one of the critical points is zero.

We will assume (for simplicity) that all the roots are distinct. In this case the behaviour of the metric and the curvature tensors near the singularities can be easily obtained from the above expressions (5.3)–(5.4). In the Jordan frame, we see that the points $z = 1/v_i$ are curvature singularities which are a finite proper distance away from other nearby points (i.e. $\int e^B dz$ does not diverge as $z \to 1/v_i$). On the other hand, the points $z = 1/u_i, 1/w_i$ are all at infinite proper distance from other points, and are not curvature singularities. They therefore correspond to spatial infinities. Hence all the roots of P_0, P_1 and $P_0Q_1 - Q_0P_1$ are either naked curvature singularities or spatial infinities. A slightly different analysis is required for $1/z \to 0$, but the same results apply. The above expressions actually give several solutions; one for each range of z which does not contain any coordinate singularities, and for which B (5.8) is real.

The situation is similar in the Einstein frame. The critical points are curvature singularities at finite proper distance if the corresponding critical point has $1/2 < u_i < 2/3$, $v_i < 1/2$, $v_i > 1$ or $w_i > 1/2$. Otherwise they are spatial infinities. The difference between the two frames arises because $\phi \to \pm\infty$ at the critical points.

We are interested in regions of spacetime which can be used to construct brane worlds with localised gravity, but which do not have any problems with naked singularities. If the zero-mode ψ_0 is constant as usual, the condition (4.13) for gravity to be localised on the brane is equivalent to require that the volume element (4.15) is finite. By substituting the solution (5.6)–(5.8) into eq. (4.15), we can determine whether it converges near each of the critical points. If it does then we can construct a suitable brane world using such a region of spacetime. The volume element (4.15) will converge near a critical point if it satisfies $1/2 < w_i < 1$, $v_i > 2$, $v_i < 1/2$ or $1/2 < u_i < 2/3$ as appropriate. Thus in the Jordan frame there are ranges of u_i and w_i for which regions of spacetime without curvature singularities and finite volume element can be found. However it should be noted that if $u_i, w_i > 1/2$ (or $v_i < 1/2$) then the string coupling, e^{ϕ}, diverges at the corresponding critical point. Furthermore the time taken (as perceived by an observer on the brane) for a photon to reach the brane from this point ($\int e^{B-A}dz$), is finite if $w_i > 1/2$, $w_i < 0$, $v_i > 2$, $v_i < 1/2$ or $1/2 < u_i < 2/3$. Hence the region of strong coupling will be visible from the brane. In the Einstein frame the conditions for a finite volume element are the same as those for a naked singularity.

If $\Lambda \neq 0$ a similar approach can be used. Asymptotic behaviour of the solutions can be obtained near the critical points, and this can be used to determine if the volume element is finite or if there are naked singularities. As for $\Lambda = 0$ there are no solutions with finite volume element and no curvature or string coupling singularities. Similar ideas have previously been used to analyse the $a = b = c = -\omega = 1$ case [38].

Hence, for $\mathcal{C} \neq 0$, it does not generally seem to be possible to obtain brane models with finite volume element and without naked singularities. This is not true for $\mathcal{C} = 0$ [38, 40], but the brane tension is then fine-tuned, as explained in section 3.

There are several cases for which the above analysis does not apply. If the polynomial q (4.5) which appears in the definition of the volume element (4.3) shares a root with P_1, the volume element will converge for a wider range of parameters. Similarly if $1 + 4\alpha q$ shares a root with one of the other polynomials. The situation will also be changed if any of P_0, P_1, or $P_0Q_1 - Q_0P_1$ share a root. However these situations will not generally arise without unnatural fine-tuning of the parameters ω, a, b and c. An exception is the point $z = \infty$. For many of the Kaluza-Klein like choices of parameters (2.7) this critical point corresponds to a root of more than one of the polynomials. However each of these cases has similar properties to the non-degenerate critical points discussed above.

6. Six-dimensional Kaluza-Klein case

Let us now focus on the case where ϕ is a moduli field of a compact sixth dimension. In the Jordan frame, putting $N = 1$ in the action (2.5) corresponds to $\omega = a = b = c = 0$. This is obviously the simplest case to consider for the action (2.1). We will now construct the general solution for these parameters.

6.1 General bulk solution

We start by considering the 6-dimensional Gauss-Bonnet anti-de Sitter black hole solution [45, 46]. The solution reads,

$$ds_6^2 = -V(r)dT^2 + r^2 dx_4^2 + \frac{dr^2}{V(r)} \tag{6.1}$$

where $V(r) = \kappa + r^2(1 \pm U)/(12\alpha)$ with

$$U = \sqrt{1 - \frac{\Lambda}{\Lambda_c} + \frac{\mu}{r^5}} \tag{6.2}$$

Here, dx_4^2 is an euclidean space of constant curvature κ, and $\Lambda_c = -5/(12\alpha)$. Note in passing that if $\Lambda = \Lambda_c$ the field equations have a degenerate solution with $e^{-B}A' = 1/\sqrt{12\alpha}$ and ϕ completely arbitrary. This corresponds to the Class I solutions found in ref. [21].

The integration constant μ is related to the 'AD energy' of the solution. Unlike ordinary Einstein gravity ($\alpha = 0$) there are two branches of solutions and both vacua are classically stable [47] to small perturbations. The solution corresponding to the $(-)$ choice of sign has a well defined $\alpha \to 0$ limit, whereas the second solution exists only if $\alpha \neq 0$. For lack of a better name we will call the $(-)$ branch the Einstein branch, and the $(+)$ branch the Gauss-Bonnet branch. Note that particular attention has to be given to the 'AD energy' of the Gauss-Bonnet branch as was shown in ref. [47] (see also [48] for a hamiltonian approach). We emphasise that the Gauss-Bonnet branch does not have an equivalent solution in Einstein theory.

Let us focus on the planar case $\kappa = 0$. For both branches there is the standard curvature singularity at $r = 0$. If $(1 - \Lambda/\Lambda_c)\mu < 0$ then there will be an additional singularity at $r = r_* = (\Lambda/\Lambda_c - 1)^{-1/5}\mu^{1/5}$, where $U = 0$. Only the solution with the lower choice of sign has an event horizon, and then only if $\Lambda\mu < 0$. The horizon is located at $r = r_h = (\Lambda_c/\Lambda)^{1/5}\mu^{1/5}$. Otherwise there is no black hole but merely a naked singularity.

A conformally flat, five-dimensional scalar field solution is obtained from the black hole (6.1) by the analytic continuation $t = ix_4$ and $X_1 \propto iT$, comparison with (2.6), and then compactification of the sixth dimension. The scalar field solution in brane background (3.1) reads,

$$A = \ln\frac{r}{r_b} + A_b\,, \tag{6.3}$$

$$\phi = -\frac{1}{4}\ln\frac{r^2(1 \pm U)}{r_b^2(1 \pm U_b)} + \phi_b\,, \tag{6.4}$$

$$B = -\frac{1}{2}\ln r^2(1 \pm U) + \frac{1}{2}\ln 12\alpha\,, \tag{6.5}$$

where the parameters A_b, ϕ_b and r_b are constants which will be convenient when discussing boundaries, and

$$U_b = U(r_b) = \sqrt{1 - \frac{\Lambda}{\Lambda_c} + \frac{\mu}{r_b^5}}\,.$$

Note that the parameter μ in the above solution is proportional to the constant $\mathcal{C}$ in (3.3).

Taking $A_b = (2/3)\phi_b$ the metric in the Einstein frame reads,

$$ds_E^2 = e^{-4\phi/3} ds^2 = \left(\frac{r}{r_b}\right)^{8/3} \left(\frac{1 \pm U}{1 \pm U_b}\right)^{1/3} dx^\mu dx^\nu \eta_{\mu\nu} + \frac{12\alpha e^{-4\phi_b/3} dr^2}{r_b^{2/3} r^{4/3} (1 \pm U)^{2/3} (1 \pm U_b)^{1/3}}\,. \tag{6.6}$$

If the original vacuum solution has an event horizon, it will be transformed into a curvature singularity for the scalar field system. This is a common phenomenon of conformally flat scalar field spacetimes obtained in this way. Of course a naked singularity signals the endpoint of the validity of a particular solution. There are a total of 7 distinct types of spacetimes which can be obtained from the metric (6.1) in this way. They can be classified according to the choice of sign in the definition of $V(r)$, the sign of μ, and the value of Λ. Most of them have naked curvature singularities which restrict the allowed range of r.

For the Gauss-Bonnet branch, if $\Lambda < \Lambda_c$ (in which case μ must be positive), the metric is only well-defined in the coordinate range $0 < r < r_*$. This solution has two naked singularities which are a finite proper distance apart. If $\Lambda > \Lambda_c$ the solution is well-defined for $r > r_*$ if $\mu < 0$, and for $r > 0$ if $\mu \geq 0$. These solutions have one naked singularity and are of infinite proper distance i.e. infinite in the extra dimension.

For the Einstein branch we have more possibilities due to the additional singular point $r = r_h$. The metric is only well-defined for $r_h < r < r_*$ if $\Lambda < \Lambda_c$ (and $\mu > 0$), and for $r_* < r < r_h$ if $\Lambda > 0$ (in which case μ must be negative). Again we have two naked singularities separated by finite proper distance. When $\Lambda_c < \Lambda < 0$ the solution is well defined for $r > r_h$ if $\mu > 0$ and for $r > r_*$ if $\mu < 0$ (or just simply $r > 0$ if $\mu = 0$). If $\Lambda = 0$ for the Einstein branch, μ must be negative and the metric may be defined for $r > r_*$, but has a naked singularity at infinity ($r \to \infty$).

Finally for $\Lambda = \Lambda_c$, there is only one branch,[3] $1 \pm U$ being replaced by $1 + \bar{\mu}/r^{5/2}$. The solution is well-defined for $r > (-\bar{\mu})^{2/5}$ if $\bar{\mu} < 0$ and for $r > 0$ if $\bar{\mu} > 0$.

For comparison with the previous section, the three $\Lambda = 0$ solutions: Gauss-Bonnet branch with $\mu < 0$, Gauss-Bonnet branch with $\mu > 0$, and Einstein branch with $\mu < 0$ respectively correspond to the three ranges $-\infty < z < -1/2$, $-1/2 < z < 1/8$ and $3/4 < z < \infty$ of the coordinate $z = d\phi/dA$. The points $z = -1/2, 3/4$ are spatial infinities ($r = \infty$), while $z = 0, 1/8$ are the curvature singularities ($r = r_*, 0$ respectively).

An infinite (proper distance) brane world solution can be constructed by taking two halves of infinite bulk spacetimes (not necessarily identical) and 'gluing' them together. The brane acts as a boundary for the bulk spacetime on both sides permitting the elimination of one curvature singularity. We see immediately that spacetimes with two naked singularities (which are also of finite proper distance) are unsuitable for constructing singularity free

[3] In $D = 5$ this case is related to Chern-Simons gravity (see for instance [49]).

brane worlds. Thus we can construct singularity free brane world models using the any of the $\Lambda \geq \Lambda_c$ Gauss-Bonnet branch solutions or the $\Lambda_c \leq \Lambda < 0$ Einstein branch solutions, provided that we keep the appropriate singularity free half of the bulk spacetime. Note that we keep asymptotic infinity $r \to \infty$ which represents proper infinity too. We take the brane to be positioned at $r = r_b$. The constants A_b and ϕ_b have to be the same on both sides of the brane, in order to guarantee a well-defined induced metric as well as continuity of the scalar field. In contrast, the parameter μ, or equivalently U_b, may take different values on each side.

Before moving on to boundary conditions we note in passing that if the black hole horizon is not flat then analytic continuation (see for instance [50] for an $\alpha = 0$ example) gives in the Einstein frame,

$$ds_E^2 = V^{1/3} r^2 (-dt^2 + e^{2\sqrt{\kappa} t} dx_3^2) + V^{-2/3} dr^2 \tag{6.7}$$

with $\phi = -\frac{1}{4} \ln V(r) + \phi_0$. Notice then how the curvature of the horizon after analytic continuation translates into the de Sitter or anti-de Sitter expansion of the brane world sections. The metric (6.7) represents the bulk background for a brane world of constant curvature. Having obtained the full set of spacetime solutions we now can move on to the boundary conditions.

6.2 Brane tension and parameter tuning

For a Z_2-symmetric brane world, the solution (6.3)–(6.5) is defined up to 2 integration constants, namely ϕ_b and μ (the constant A_b is not physical and may be taken to vanish), and 2 possible branches. Applying the junction conditions (3.6) and (3.7) to this solution, and taking the brane tension to be $T(\phi) = T_1 e^{-\chi\phi}$, we find

$$\chi = 2 + \frac{30\mu}{(U_b^2 \mp 16U_b - 32 + 15U_0^2) r_b^5} \tag{6.8}$$

and

$$T_1 = -\frac{M^3}{\sqrt{12\alpha}} \frac{32 \pm 16U_b - U_b^2 - 15U_0^2}{6(1 \pm U_b)^{1/2}} e^{(\chi-2)\phi_b} = \frac{5\mu M^3}{\sqrt{12\alpha} r_b^5 (\chi - 2)\sqrt{1 \pm U_b}} e^{(\chi-2)\phi_b} \tag{6.9}$$

A few comments are now in order. If $\mu = 0$ then $\chi = 2$ and therefore T_1 must be fine-tuned [40]. Otherwise fixing χ, the dilaton coupling to the brane, fixes the value of the integration constant μ for given Λ. Then, the brane tension parameter T_1 in eq. (6.9) still depends on the arbitrary integration constant ϕ_b in eq. (6.4), which could conceivably tune itself to allow four-dimensional flat solutions for any values of the tension. This is the basic idea behind 'self-tuning'. It is also easy to see that the brane tension can be positive for both branches.

If we relax the assumption of Z_2-symmetry across the brane, the parameter μ may take different values on each side of the brane, and so we will have one extra arbitrary integration constant in the expressions for χ and T_1.

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6.3 Stability and four-dimensional effective gravity

The functions appearing in the graviton equations (4.3)–(4.6) are:

$$w = \mp\sqrt{3\alpha}\frac{r^2}{r_b^3}\frac{50U^2U_0^2 - U^4 - 25U_0^4}{12U^3\sqrt{1\pm U_b}}\, e^{2A_b-2\phi_b} \tag{6.10}$$

$$p = \mp\frac{r^6}{r_b^5}\frac{(1\pm U)(U^2+5U_0^2)}{12U\sqrt{3\alpha}\sqrt{1\pm U_b}}\, e^{4A_b-2\phi_b} \tag{6.11}$$

$$v = \mp\frac{r^3}{r_b^3}\frac{3U^2 \pm 8U + 5U_0^2}{8\sqrt{3\alpha}U\sqrt{1\pm U_b}}\, e^{2A_b-2\phi_b} \tag{6.12}$$

where $U_0 = U(\mu = 0) = \sqrt{1-\Lambda/\Lambda_c}$ and the lower sign corresponds to the Einstein branch. In these coordinates r ranges from r_b to ∞. We will only consider the cases without naked bulk singularities ($U_0 \geq 0$ for the Gauss-Bonnet branch, and $0 \leq U_0 < 1$ for the Einstein branch).

First note that the bulk graviton equation (4.10) is singular in the limit $r \to \infty$, since the functions w (6.10) and p (6.11) blow up. This is always true except for the trivial case of the constant zero mode. In the Schrödinger formulation (4.16), this translates into a potential $V(x)$ diverging at a finite distance from the brane in the x-coordinate. The situation is thus similar to that of a particle confined in a box, and we shall impose that the wave-functions Φ_m vanish "at the edge of the box". This is equivalent to require:

$$\psi_m \sim o\left(\frac{1}{r^2}\right) \qquad \text{for } r \to \infty\,. \tag{6.13}$$

All the modes satisfying eq. (6.13) are normalisable on the bulk interval with respect to the weighting function $w(r)$ (6.10). Considering the asymptotic behaviour of the wave equation (4.10) with the expressions (6.10) and (6.11) shows that eq. (6.13) selects a one-parameter family of solutions for ψ_m on each side of the brane. Continuity on the brane and the junction conditions (4.11) then result in a homogeneous system of two equations for the two unknown parameters. Vanishing of its determinant then gives the allowed values for m^2, which are therefore discrete. Since α is the only other mass scale appearing in the problem, we expect the order of magnitude of mass spacings between the modes to be:

$$\Delta m \sim \frac{1}{\sqrt{\alpha}}\,. \tag{6.14}$$

Secondly note from eq. (6.11) that the sign of p is always negative for the Gauss-Bonnet branch. This implies that the sign of the kinetic term in an effective action for $\bar{\gamma}_{\mu\nu}$ is also negative. This branch therefore suffers for a graviton ghost in the bulk. The same is true for the corresponding original six-dimensional black-hole vacuum [45], although it has been argued to be classically stable [47]. On the other hand, p is always positive for the Einstein branch, so this case is ghost free. As discussed in section 4, there may be tachyon modes if either w or $[v]$ is negative. This could occur for the Einstein branch, in which case the solution would be unstable.

To illustrate the above two points consider the case $\mu = 0$. We can then find analytic expressions for the modes. Indeed define:

$$\lambda_{\pm}^2 = |m^2| \frac{12\alpha}{1 \pm U_0} r_b^2 \, e^{-2A_b} \,. \tag{6.15}$$

For $m^2 > 0$ the bulk solution satisfying (6.13) is

$$\psi_m \propto \frac{1}{r^{5/2}} \, J_{5/2} \left(\frac{\lambda_{\pm}}{r} \right) \tag{6.16}$$

and the mass spacings between the Kaluza-Klein states are given by:

$$\Delta m^2 \sim \frac{1 - U_0}{12\alpha} \, e^{A_b} \tag{6.17}$$

in agreement with (6.14).

Now consider a solution with $m^2 < 0$, satisfying (6.13) at infinity,

$$\psi_m \propto \frac{1}{r^{5/2}} \, I_{5/2} \left(\frac{\lambda_{\pm}}{r} \right) . \tag{6.18}$$

In the case of two Einstein branches on each side of the brane, the corresponding junction condition (4.11) then implies

$$\frac{\lambda_-}{r_b} \, I_{5/2} \left(\frac{\lambda_-}{r} \right) = \frac{3U_0}{4(1 - U_0)} \, I_{3/2} \left(\frac{\lambda_-}{r} \right) \tag{6.19}$$

If $0 < U_0 < 1$, this equation is always satisfied by one real value of λ_-, and so there is always a tachyon mode when $\mu = 0$ for the Einstein branch, even though p and w are positive. For small U_0, $m^2 \sim -(U_0/\alpha)e^{2A_b}/r_b^2$. The higher order curvature term has destabilised the solution. Note in contrast that the junction conditions do not allow for any tachyonic gravitons in the case where the Gauss-Bonnet branch is present on at least one side of the brane (and $\mu = 0$).

To ensure that $w > 0$ we require $5(5 - 2\sqrt{6})U_0^2 < U_b^2 < 5(5 + 2\sqrt{6})U_0^2$. To ensure $[v] \geq 0$ we need $U_b \leq (4/3)(1 - \sqrt{1 - 15U_0^2/16})$, and so μ must be negative. It is only possible to satisfy both these conditions simultaneously if $U_0 \gtrsim 0.87$, hence Λ/Λ_c must be small if the spacetime is to be free from tachyons and ghosts.

As for the constant zero mode, we see from eq. (6.10) that the volume element (4.15) is infinite for the solution (6.3)–(6.5), in agreement with the general discussion of section 5. In the present example this is because, in order to avoid naked singularities, we have kept the region of space-time which contains the conformal infinity of the original six-dimensional space (6.1), which is asymptotically anti-de Sitter. Nevertheless, it is possible to localise gravity on the brane, if we break Z_2 symmetry by joining together an Einstein with a Gauss-Bonnet branch. The price to pay however is a bulk graviton ghost on the Gauss-Bonnet side. Indeed, for $m^2 = 0$, the solution (4.14) of the perturbation equation (4.10) which satisfies the boundary condition (6.13) is given by:

$$\psi_0 = C \left[\pm \frac{5U_0^2}{2} \ln \left(\frac{U^2}{6U_0^2} + \frac{5}{6} \right) \pm \ln \left(\frac{1 \pm U}{1 \pm U_0} \right) - \sqrt{5} U_0 \, \arctan \frac{\sqrt{5}(U - U_0)}{U + 5U_0} \right] \tag{6.20}$$

for $\mu \neq 0$ and C/r^5 if $\mu = 0$, where the constant C may be different on both sides of the brane. The junction condition (4.11), together with continuity of ψ_0 on the brane, then require $\mu \neq 0$ and two different branches of the solution on each side of the brane. The special junction conditions for higher order terms discussed in section 4, as well as the two possible signs of p (and hence the ghost) are essential here. In this case, the zero-mode (6.20) is normalisable, and separated from the massive ones by the mass gap (6.14). Since it is natural for α to be of the order of the fundamental scale of the underlying theory, the contribution of the massive gravitons to brane gravity at low energies is likely to be negligible. This setup is still free of singularities in the bulk, and the tension of the brane is not fine-tuned.

7. Conclusions

In this paper we have investigated the properties of a Poincaré brane world, located in a five-dimensional background containing a scalar field. We considered the most general quartic order action which produced field equations that were linear in second order derivatives, and contained no higher derivatives. If the scalar field plays the role of the dilaton, the couplings of the dilatonic higher order terms in the lagrangian (2.1) are constrained in order to reproduce the scattering string amplitudes on shell [34]. We have also derived their values (2.7) when the scalar field arises as the unique modulus coming from the Kaluza-Klein toroidal compactification of N extra dimensions.

In order to study how gravity may be perceived by a four-dimensional observer, we considered background perturbations corresponding to graviton modes which are tangent to the brane. Although the tensorial structure of bulk gravitons is still the same as in Einstein gravity, the higher order terms lead to significant modifications to the original mechanism of gravity localisation. In particular, they lead to mixed rather than Neumann boundary conditions (4.11), much like induced gravity models [7], where the momenta of modes appear in the boundary conditions themselves. This renders questions of eigenmode orthogonality and of background stability far more subtle and constrained than the Einstein case (4.12). For example it is possible to have a scalar field background which is stable in Einstein gravity, but which develops tachyon modes when a Gauss-Bonnet correction is included. A solution with this type of instability (6.18) was described in the previous section.

Furthermore, for some non Z_2-symmetric configurations, gravity may be localised on the brane, even though the volume element is infinite. This involves a non-constant graviton zero mode, and is only possible if there are multiple branches of solutions, as occurs in Einstein-Gauss-Bonnet gravity. However, combining different branches in this way generically implies a bulk graviton ghost on one side of the brane.

In section 5 we presented a method for analytically obtaining the general solution of the field equations, which as we showed in section 3, is a necessary step if one is to address the 'self-tuning' mechanism in this context (i.e. trying to resolve the four-dimensional cosmological constant problem by allowing four-dimensional Poincaré invariance on the brane to be obtained without fine-tuning of its tension, due to its conformal coupling to

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the scalar field [30]). We then argued that requiring finite volume element, as well as no naked singularities (and finite string coupling), generally implies fine-tuning rather than self-tuning of the brane tension.

Finally, to illustrate all of the above, we considered in detail the case where the scalar field is the modulus of a sixth flat dimension. The general solution then is simply obtained by analytic continuation of the corresponding six-dimensional Gauss-Bonnet black-hole [45, 46]. The spatial topology of the black hole horizon transforms into the four-dimensional spacetime curvature of the brane thus providing the de Sitter, Poincaré, and anti-de Sitter brane solutions. Regular brane solutions can then be constructed without fine-tuning of tension, or naked singularities in the bulk. It is rather intriguing that the zero-mode graviton is generically non-normalisable i.e. gravity is not localised, except if we consider different branches of solutions on each side of the brane. Such a setup allows for a normalisable zero-mode graviton to be separated from the (discrete) massive ones by a mass gap, of the order of the fundamental scale coupling of the higher order terms. The drawback of this case is the presence of a bulk graviton ghost in the Gauss-Bonnet branch which signals a potential quantum inconsistency of the gravitational background. This is a rather subtle problem since the corresponding original six-dimensional black-hole anti-de Sitter vacuum [45] has also been argued to be classically stable [47] to small perturbations due to the change of sign of the AD energy in the Gauss-Bonnet branch. The presence, significance and eventual cure of such bulk ghosts certainly demands further study.

Furthermore, on a different note, higher dimensional toroidal compactification should relate the scalar field solutions discussed here (for $N > 2$) to the Gauss-Bonnet equivalent of cosmic p-brane solutions [52]. Such solutions yield backgrounds of higher co-dimension brane worlds [53, 54]. It would be intriguing to know their Gauss-Bonnet versions and to see if they can solve some of the problems of their Einstein counterparts such as the presence of the bulk naked singularity or the fate of self-tuning in higher co-dimension spacetimes (see for example [55]).

Acknowledgments

It is a great pleasure to thank Pierre Binetruy, Brandon Carter, Jihad Mourad, Luigi Pilo, Valery Rubakov and Dani Steer for encouraging and discouraging remarks and observations. CC thanks G. Kofinas for discussions on Chern-Simmons gravity. SCD is grateful to the EC network HPRN–CT–2000–00152 for financial support.

A. Field equations

Varying the action (2.1) with respect to the metric gives:

$$
\begin{aligned}
f\Lambda g_{ab} + fG_{ab} - f'\left[\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right] - f''\left[\nabla_b\phi\nabla_a\phi - g_{ab}(\nabla\phi)^2\right] - \\
- 4\omega f\left(\nabla_b\phi\nabla_a\phi - \frac{1}{2}g_{ab}(\nabla\phi)^2\right) + 2\alpha\left(fH_{ab} + 2P_{aebc}(f''\nabla^e\phi\nabla^c\phi + f'\nabla^e\nabla^c\phi)\right) -
\end{aligned}
$$

$$
\begin{aligned}
&-8b\alpha\Big(f\Big[2\nabla^e\phi\nabla^c\phi P_{aebc}+(\nabla\phi)^2G_{ab}+2(\nabla_a\nabla_e\phi)(\nabla_b\nabla^e\phi)-\\
&\quad -2(\nabla_a\nabla_b\phi)\Box\phi+(\Box\phi)^2g_{ab}-(\nabla^e\nabla^c\phi)(\nabla_e\nabla_c\phi)g_{ab}\Big]+\\
&\quad +f'\Big[(\nabla\phi)^2\Box\phi g_{ab}-(\nabla^e\nabla^c\phi)\nabla_e\phi\nabla_c\phi g_{ab}-\nabla_a\phi\nabla_b\phi\Box\phi-\\
&\quad -(\nabla\phi)^2\nabla_a\nabla_b\phi+2\nabla^e\phi\nabla_{(a}\phi\nabla_{b)}\nabla_e\phi\Big]\Big)+\\
&+16a\alpha\Big(f\left[(\nabla^e\phi\nabla^c\phi(\nabla_e\nabla_c\phi)g_{ab}-2\nabla^e\phi\nabla_{(a}\phi(\nabla_{b)}\nabla_e\phi)+\nabla_a\phi\nabla_b\phi\Box\phi\right]+\\
&\quad +\frac{1}{2}f'\left[(\nabla\phi)^2g_{ab}-2(\nabla\phi)^2\nabla_a\phi\nabla_b\phi\right]\Big)-\\
&-16c\alpha f\left(2(\nabla\phi)^2\nabla_a\phi\nabla_b\phi-\frac{1}{2}(\nabla\phi)^4g_{ab}\right)=0
\end{aligned}
\tag{A.1}
$$

where the prime denotes differentiation with respect to ϕ, and

$$
H_{ab}=R_{adec}R_b{}^{dec}-2R^{ec}R_{eacb}-2R_{ae}R^e{}_b+RR_{ab}-\frac{1}{4}g_{ab}\mathcal{L}_{\rm GB}
\tag{A.2}
$$

$$
P_{aebc}=R_{aebc}+R_{be}g_{ac}+R_{ac}g_{be}-R_{ec}g_{ab}-R_{ab}g_{ec}+\frac{1}{2}Rg_{ab}g_{ec}-\frac{1}{2}Rg_{be}g_{ac}\,.
\tag{A.3}
$$

Varying (2.1) with respect to the scalar field gives:

$$
\begin{aligned}
0=&-8\omega f\Box\phi-4\omega f'(\nabla\phi)^2-f'R+2f'\Lambda+2\Lambda' f+\\
&+\alpha\Big(-f'\mathcal{L}_{\rm GB}-16(3cf'+af'')(\nabla\phi)^4-16bG^{ab}\left(f'\nabla_a\phi\nabla_b\phi+2f\nabla_a\nabla_b\phi\right)+\\
&+32af(\Box\phi)^2-64cf(\nabla\phi)^2\Box\phi-64(2cf+af')\nabla^b\phi\nabla^a\phi\nabla_b\nabla_a\phi-\\
&-32af\left(\nabla^a\nabla^b\phi\nabla_a\nabla_b\phi+R^{ab}\nabla_a\phi\nabla_b\phi\right)\Big)
\end{aligned}
\tag{A.4}
$$

For a conformally flat ansatz of the form (3.1), the (μ,ν) and (5,5) components of (A.1) read, respectively:

$$
\begin{aligned}
&4\phi''-6A''-4(\omega+2)\phi'^2-12A'^2-2\Lambda e^{2B}+12A'\phi'+6A'B'-4\phi'B'+\\
&+24\alpha e^{-2B}\left(A'^2A''-2A'^2\phi''-4\phi'A'A''+4A'^2\phi'^2-6\phi'A'^3+A'^4-A'^3B'+6A'^2B'\phi'\right)+\\
&+48b\alpha e^{-2B}\left(\phi'^2A''+2A'\phi'\phi''+2A'^2\phi'^2-2A'\phi'^3-3A'B'\phi'^2\right)+\\
&+32a\alpha e^{-2B}\left(-\phi'^2\phi''+\phi'^4+B'\phi'^3\right)-16c\alpha e^{-2B}\phi'^4=0
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
&e^{2B}2\Lambda-4\omega\phi'^2+12A'^2-16\phi'A'+\\
&+\alpha e^{-2B}\left(24[-A'^4+8A'^3\phi']-288b[A'^2\phi'^2]+32a[4A'\phi'^3+\phi'^4]-48c[\phi'^4]\right)=0
\end{aligned}
\tag{A.6}
$$

while the scalar field equation (A.4) gives:

$$
\begin{aligned}
&-4\omega\phi''-8A''+4\omega\phi'^2-20A'^2-2\Lambda e^{2B}-16\omega A'\phi'+8A'B'+4\omega\phi'B'+\\
&+\alpha e^{-2B}\Big(24\left[4A'^2A''+5A'^4-4A'^3B'\right]-\\
&-96b\left[A'^2\phi''+2\phi'A'A''-A'^2\phi'^2+4\phi'A'^3-3A'^2\phi'B'\right]\Big)+\\
&+32a\alpha e^{-2B}\left(2\phi'^2A''+2\phi'^2\phi''+4A'\phi'\phi''+8A'^2\phi'^2-\phi'^4-6A'B'\phi'^2-2B'\phi'^3\right)-\\
&-16c\alpha e^{-2B}\left(6\phi'^2\phi''+8A'\phi'^3-3\phi'^4-6B'\phi'^3\right)=0
\end{aligned}
\tag{A.7}
$$

where a prime denotes derivative with respect to z.

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